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# THE AESD PARABOLIC EQUATION MODEL

H. K. BROCK

Numerical Modeling Division
Naval Oceanographic Laboratory

January 1978



NAVAL OCEAN RESEARCH AND DEVELOPMENT ACTIVITY
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# FOREWARD

Numerical models for calculating ocean sound transmission loss continue to be of high interest, especially those which are useful for calculations in an environment that changes with range. The model described in this report is a powerful tool for application in such environments.

When this report was prepared, the author worked for the Acoustic Environmental Support Detachment (AESD) of the Office of Naval Research. Prior to publication, the function of AESD was incorporated into the Numerical Modeling Division (Code 320) of the Naval Ocean Research and Development Activity (NORDA). The report's contents are of sufficient interest and general applicability to warrant making them available by publication at this time as a NORDA technical note. The original preparation of the material was sponsored by the Long Range Acoustic Propagation Project of ONR (now NORDA Code 600).

C. G. DARRELL, CAPT, USN
Commanding Officer, NORDA

## ABSTRACT

The Parabolic Equation Model is a wave-acoustics model designed for the computation of acoustic transmission loss as a function of range and depth in range dependent ocean environments. The elliptic reduced wave equation is approximated by a parabolic partial differential equation that can be numerically integrated by marching the solution forward in range. The model is primarily useful for predicting low frequency (< 200Hz) acoustic propagation of energy along waterborne paths. This report briefly describes the physics and mathematics of the model and documents a computer program developed at AESD. Individual routines are documented in an appendix. Environmental input routines must be supplied by the user and are not described in this report.

## CONTENTS

THE AESD PARABOLIC EQUATION MODEL

## I INTRODUCTION

The Parabolic Equation (PE) Model replaces the reduced elliptic wave equation with a parabolic partial differential equation that can be integrated numerically using the Tappert-Hardin split-step Fourier algorithm. This parabolic approximation is useful when propagation is predominantly radial and backscatter can be neglected. The parabolic wave equation includes diffraction and all other full-wave effects as well as range dependent environments. The numerical algorithm has exponential accuracy in depth, second order accuracy in range and is unconditionally stable. The entire range and depth dependent acoustic field is computed as the solution is marched forward in range. The model assumes a flat pressure release ocean surface and a vanishing field at the maximum depth of the finite Fourier transform. A pseudo radiation condition is introduced at the water-bottom interface by smoothly attenuating the field. Since the error in the parabolic approximation increases as angles increase from the horizontal, steep slopes can cause inaccuracies. Also, computing time increases as

frequency increases. Thus, the model is primarily useful for predicting low-frequency " $<$ 200Hz" acoustic propagation of energy along waterborne or shallow angle bottom-bounce paths. In addition the numerical solution is limited by practical Fourier transform sizes and computing time. Therefore the frequencies that can be handled by the model depend on the input environment.

## II THE PARABOLIC APPROXIMATION

In a medium of constant density, cylindircally symmetric about an axis containing a point time-harmonic source, the acoustic pressure P satisfies the reduced elliptic wave equation

$$\left[\nabla^2 + K_o^2(r,z)\right] P(r,z) = 0 \qquad (1)$$

where $K_o$ is a reference wave number

$$K_o = \frac{\omega}{C_o} \qquad (2)$$

$\omega$ is the angular frequency of the source, $C_o$ is a reference sound speed, and n is the index of refraction

$$n(r,z) = \frac{C_o}{c(r,z)} \qquad (3)$$

Writing the acoustic pressure in the form

$$P(r,z) = \psi(r,z)\ H_o^{(1)}(K_o r) \qquad (4)$$

where the Hankel function $H_o^{(1)}$ represents the primary radial dependence of the field in terms of an outward propagating cylindrical wave, and under the assumption that the observation range, r, is many wavelengths from the source (i.e. $K_o r >> 1$), we may make use of the asymptotic form

$$H_O^{(1)}(K_O r) \sim \left(\frac{2}{\pi K_O r}\right)^{1/2} e^{i(K_O r - \pi/4)} \tag{5}$$

to obtain

$$\psi_{rr} + 2iK_O\psi_r + \psi_{zz} + K_O^2[n\ (r,z)-1]\psi = 0 \tag{6}$$

Up to this point we have made cylindrical symmetry and far field approximations. We now consider propagation that is predominantly radial and make the additional "parabolic" approximation

$$\psi_{rr} << 2iK_O\ \psi_r \tag{7}$$

which is equvalent to neglecting the reflected field and are left with the Leontovich-Fock[1] parabolic equation for the transmitted field

$$\psi_r = i\ (A+B)\ \psi \tag{8}$$

where A and B are the real operators

$$A = \frac{1}{2K_O}\ \frac{\partial^2}{\partial z^2} \tag{9}$$

$$B = \frac{K_O}{2}\ (n^2-1) \tag{10}$$

and

$$|P|^2 = \frac{1}{r}\,|\psi|^2 \qquad (11)$$

(There is a constant of proportionality to be determined for $\psi$ wnich is related to the source strength normalization. See Section 3.2.) This approximation was first used in underwater acoustic applications by Tappert and Hardin.[2]

McDaniel[3] examined the effect of this parabolic approximation on the propagation of normal modes in a laminar ocean and has shown that discrete modes are propagated with the correct amplitudes and mode shapes, but with errors in phase and group velocities. This error can cause substantial shifts in the modal interference pattern. In general the solution of the parabolic equation well approximates the solution of the elliptic wave equation (equation (1)) over a narrow region of the spectrum of eigenvalues. The eigenmode with range phase velocity corresponding to the reference sound speed is propagated correctly. Hence the parabolic approximation can be described as a "narrow-band" approximation.[4]

In many applications where a single group of modes or in the ray analogy, a single ray family, dominates the propagation, the parabolic approximation is entirely

adequate. One simply selects the proper turning point velocity as the reference sound speed. However, when many modes are propagating such a choice is not possible and the resultant errors may be unacceptable.

As a means of improving the narrowband parabolic approximation, Buchal[5] has suggested a mapping of the index of refraction that approximately preserves the phase velocities and turning points of the normal modes with turning points in the water column. The sound speed profile is perturbed to produce a modified index of refraction such that the phase velocities of the parabolic solution modes equal the phase velocities of the elliptic solution modes. The WKB approximation is then used to map the mode turning points. The resultant mapping

$$(z,n) \rightarrow (\tilde{z},\tilde{n}) = \left(zn^{1/2},\ (zn-1)^{1/2}\right) \tag{12}$$

significantly improves the agreement between the elliptic and parabolic solutions.

## III THE NUMERICAL ALGORITHM

### A. Description

For a useful long-range low-frequency acoustic propagation model we require the numerical solution of equation 8 for realistic range dependent ocean environments. Several numerical methods are applicable to the solution of equations of this form. We have chosen to implement the split-step Fourier method of Tappert and Hardin.[6] This algorithm is applicable to a particular class of equations and has several advantages. In addition to exponential accuracy in Z and second order accuracy in r, it is exactly energy conserving, unconditionally stable, and computationally efficient. The algorithm does, however, require, a uniform mesh and periodic boundary conditions in Z because of its use of the fast Fourier transform. The sound speed profiles must also be filtered to avoid introducing high angular frequency components into the acoustic field. This means that discontinuities must be smoothed.

Following the analysis of Buchal and Tappert,[7] a solution of equation 8 can be formally represented as

$$\psi(r+\Delta r,z) = \left[ e^{i\int_r^{r+\Delta r} U(\tilde{r})\,d\tilde{r}} + E \right] \psi(r,z)$$

$$\approx \left\{ e^{i\Delta r U} - \frac{(\Delta r)^3}{12}\left[2(UU_r+U_rU) + i\,U_{rr}\right] + 0\left((\Delta r)^4\right)\right\} \psi(r,z) \tag{13}$$

where $U = A+B$

$A$ and $B$ are defined by equations 9 and 10

and $U$, $U_r$, $U_{rr}$ are evaluated at $r + \frac{\Delta r}{2}$.

Neglecting terms of third and higher orders, the numerical algorithm becomes

$$\psi(r+\Delta r,z) = e^{i\Delta r(A+B)}\,\psi(r,z) \tag{14}$$

The implementation of the split-step algorithm requires the assumption that the operators A and B commute, which they do not. If the operator is written as follows

$$\begin{aligned} e^{i\Delta r(A+B)} &= e^{\frac{i\Delta rA}{2}}\, e^{i\Delta rB}\, e^{\frac{i\Delta rA}{2}} \\ &\quad - \frac{i(\Delta r)^3}{6}\Big\{\frac{1}{4}\big[A(AB-BA) - (AB-BA)B\big] \\ &\qquad + \frac{1}{2}\big[B(AB-BA) - (AB-BA)B\big] \\ &\quad + 0\Big((\Delta r)^4\Big) \end{aligned} \tag{15}$$

then, again to third order in $\Delta r$, the numerical algorithm takes the form

$$\psi(r+\Delta r,z) = e^{\frac{i\Delta rA}{2}}\, e^{i\Delta rB}\, e^{\frac{i\Delta rA}{2}}\,\psi(r,z) \tag{16}$$

The evaluation of $e^{i\Delta rB}$ is a straight-forward multiplication. Since A is a differential operator $e^{i\Delta rA}$ is evaluated by means of a fast Fourier transform operation. Thus the form of the split-step algorithm implemented here becomes

$$\psi(r+\Delta r,z) = e^{\frac{i\Delta r K_o}{2}(n^2-1)} \; FFT^{-1}\left[ e^{\frac{-i\Delta r K^2}{2\,K_o}} FFT\left(\psi(r,z)\right)\right] \qquad (17)$$

where FFT is the spatial Fourier transform from z to K and $FFT^{-1}$ is the inverse transform. This algorithm advances the field by alternating between two stages. The first stage advances the field as if the propagation were in a homogeneous medium thus accounting for the effects of diffraction. The second stage then accounts for the effects of the environment on the propagation.

## B. Implementation

The essence of the parabolic equation technique is the numerical solution of the Leontovich-Fock parabolic wave equation by the Tappert-Hardin split step Fourier algorithm, subject to appropriate initial and boundary conditions. To satisfy the pressure release boundary condition at the ocean surface

$$\psi(r,o) = 0 \qquad (18)$$

$\psi(r,z)$ is made anti-symmetric in z. This is accomplished by replacing the complex Fourier transform operations in equation 17 by real sine transform operations and transforming the real and imaginary parts of the field separately. A pressure release surface at the maximum depth sample considered in the transform is, however, implicit in this procedure. This non-physical boundary condition is handled by the following artifice: We extend the depth sampling in the transform 1/4 the maximum physical depth relevant to the problem and assume an index of refraction in this region of the form

$$n^2 = n_B^2 + i\alpha e^{-\left(\frac{z-z_{max}}{\beta}\right)^2} \tag{19}$$

where $n_B$ is the index of refraction at the water-bottom interface and $z_{max}$ is the maximum depth sample in the transform. This form attenuates the additional discrete modes introduced by truncating the transform and, with proper choice of $\alpha$ and $\beta$, models a pseudo radiation condition at the water bottom interface. The constants $\alpha$ and $\beta$ were chosen empirically to give agreement with a normal mode program for the ocean index of refraction over a homogeneous half-space. The values found by this process are

$\alpha = .01$

$\beta = 1/3$ the artificial bottom depth.

Before the split-step algorithm can be used to march the field forward, the field must be defined over the entire depth mesh as an initial condition at some range. In addition, the numerical method requires that this field be band limited. In the implementation considered here, the initial field is assumed to be a Gaussian beam

$$\psi(0,z) = S\, e^{\frac{-(z-Z_O)^2}{W^2}} \tag{20}$$

where S is an effective source level, W is a beamwidth, and $Z_O$ is the depth of the point source being modeled. To evaluate S and W we solve the parabolic wave equation in a homogeneous medium

$$2iK_O\psi_r + \psi_{zz} = 0 \tag{21}$$

with equation 20 as an initial condition obtaining

$$\psi(r,z) = S\left[1+\frac{i2r}{K_O W^2}\right]^{-1/2} e^{-\frac{(z-Z_O)^2}{W^2\left(1+\frac{i2r}{K_O W^2}\right)}} \tag{22}$$

From equation 11

$$|P|^2 = \frac{1}{r}\,|\psi|^2$$

$$= \frac{K_o W^2 S^2}{2r^2}\left[1+\frac{K_o^{\,2} W^4}{4r^2}\right]^{-1/2} e^{-\dfrac{K_o^{\,2} W^2 (Z-Z_o)^2}{2r^2\left(1+\dfrac{K_o^{\,2} W^4}{4r^2}\right)}} \tag{23}$$

$$= \frac{K_o W^2 S^2}{2r^2}\left[\left(1+\frac{K_o^{\,2} W^4}{4r^2}\right)^{-1/2} - \frac{K_o^{\,2} W^2}{2}\,\frac{(Z-Z_o)^2}{r^2}\left(1+\frac{K_o^{\,2} W^4}{4r^2}\right)^{-3/2} + \ldots\right]$$

while from the solution of the elliptic wave equation for a point source in a homogenous medium

$$|P|^2 = \frac{1}{r^2\left[1+\dfrac{(Z-Z_o)^2}{r^2}\right]}$$

$$= \frac{1}{r^2}\left[1-\frac{(Z-Z_o)^2}{r^2}+\ldots\right] \tag{24}$$

Matching the parabolic and elliptic wave equation results for large r yields

$$S = \frac{1}{W}\left(\frac{2}{K_o}\right)^{1/2} \tag{25}$$

for the effective source level of the beam and

$$W^2 = \frac{2}{K_o^2} \tag{26}$$

for the width.

The parabolic phase velocity correction (equation 12) is implemented as follows:

1. A uniform mesh is established in $\tilde{z}$ the transformed depth variable.

2. The sound speed is defined to be piecewise linear in z

$$C(z) = C_1 + g\ (z - z_1)$$

3. The mapping

$$Z_j = \tilde{Z}_j \left( \frac{C_o}{C(Z_j)} \right)^{1/2}$$

implies

$$Z_j = \frac{g\tilde{Z}_j^{\,2} + \tilde{Z}_j \sqrt{(g\tilde{Z}_j)^2 + 4C_o\ (C_1 - gZ_1)}}{2C_o}$$

and

$$n_j = \frac{C_o}{C_1 + g(Z_j - Z_1)}$$

where $C_o$ is the reference sound speed

4. The transformed index of refraction on the uniform mesh is given by

$$\tilde{n}_j = (2n_j - 1)^{1/2}$$

A 1-2-1 filter is then applied to the uniform mesh ($\tilde{z}$, $\tilde{n}$) (or ($Z$,n) if no parabolic phase velocity correction is to be applied) to smooth the gradient discontinuities associated with the boundaries in the piecewise linear sound speed profile.

Two forms of error control are employed during the march. The range step is constrained by requiring the ratios of the error terms (third order) in equation 13 and equation 15 to the first order terms be small. To evaluate these terms we approximate the reduced pressure locally as a plane wave

$$\psi(r,z) \simeq e^{iK_o[r(\cos\theta - 1) + z\sin\theta]} \tag{27}$$

Equation 14 then yields for the range integration error term

$$\Delta r << 24 \left| \frac{(n^2 - 1 - \sin^2\theta)^2 + \dfrac{(n^2)_{zz}}{K_o^2} + i\,\dfrac{2(n^2)_z}{K_o}}{K_o(n^2)_{rr} + (n^2)_{zzr} + i\,K_o\,(n^2)_{rz}\sin\theta} \right| \tag{28}$$

For $n^2$ independent of r this constraint does not limit $\Delta r$ and has not been implemented in the current version of the computer code.

Similarly from equation 15 we have for the first order term

$$i\Delta r(A+B)\psi = \frac{iK_o}{2}\Delta r(n^2-1-\sin^2\theta)\psi \tag{29}$$

and for the third order term

$$\frac{i(\Delta r)^3}{6}\left\{ {}^1/_4\left[A(AB-BA)-(AB-BA)A\right] + {}^1/_2\left[B(AB-BA)-(AB-BA)B\right]\right\}$$

$$= \frac{iK_o}{48}(\Delta r)^3\left\{((n^2)_z)^2 + {}^1/_4\left[\frac{(n)^2{}_{zzzz}}{K_o{}^2} - 4(n^2)_{zz}\sin^2\theta + i\,\frac{4(n^2)_{zzz}\sin\theta}{K}\right]\right\} \tag{30}$$

Thus our constraint becomes

$$(\Delta r) \ll \frac{24(n^2-1-\sin^2\theta)}{\left((n^2)_z\right)^2 - (n^2)_{zz}\sin^2\theta} \tag{31}$$

where we define the "angle" $\theta$ by

$$\tan\theta = \frac{1}{K_o}\left\|\frac{K\ FFT(\psi)}{FFT(\psi)}\right\| \tag{32}$$

K is the spacial Fourier transform variable from Z to K, and the $(n^2)_{zzzz}$ and $(n^2)_{zzz}$ terms have been dropped.

The depth mesh increment is constrained by requiring the initial mesh spacing to be less than the "width" of

the Gaussian beam initial condition. As the solution is marched forward, the spectrum is partitioned into 4 equal bins. The energy $E_j$ in each bin is computed and the following energy ratios are examined:

$$R_1 = \frac{E_4}{E_1 + E_2 + E_3 + E_4}$$

$$R_2 = \frac{E_3 + E_4}{E_1 + E_2}$$

$$R_3 = \frac{E_2}{E_1}$$

1. If $-20\text{dB} < R_1 < -14\text{dB}$ a transform aliasing warning is issued. Five such warnings will terminate the run.

2. If $R_1 \geq -14\text{dB}$ abiasing is considered severe and the run is immediately terminated.

3. If $R_2 < -70\text{dB}$ and $R_3 < -60\text{dB}$, the field is considered to be oversampled tnd the transform size is reduced.

Our final numerical algorithm then consists of the following steps:

1. Fourier sine transform $\psi$

2. Compute the energy ratios and the "RMS angle" $\theta$ .

3. Compute a $\Delta r_{new}$ at each mesh point using the range step constraint and search for the minimum.

4. Apply the operator

$$e^{\frac{i(\Delta r_{old} \quad \Delta r_{new})}{2} A} \psi$$

5. Fourier sine transform $\psi$

6. Apply the operator

$$e^{i\Delta r_{new} B} \psi$$

7. Advance the range $r = r_{old} + \Delta r_{new}$

## IV USE AND LIMITATIONS OF THE MODEL

Most of the limitations of the parabolic equation underwater acoustic propagation model are imposed by the acoustic frequency or are associated with modeling the ocean bottom. The assumption that the propagation is at small angles to a preferred direction in a medium with slowly varying inhomogenities is implicit in the parabolic approximation (equation 7). In addition other limitations are imposed by the numerical algorithm used here. It is evident from the third order term (equation 30) that the allowable range step is limited by the sound speed gradients. Large gradients such as those found in bottom sediment layers imply small range steps and can lead to unacceptable computation times. The computer code will issue a warning if the predicted range step is less than the acoustic wavelength. Five such warnings will cause termination of the calculation.

A small sound speed gradient approximation is also made in the solution of the integral equation that leads to the parabolic phase velocity correction mapping (equation 12). Hence the mapping presented here is applicable only to modes with turning points in the water column.

Since the required depth mesh spacing is related to the acoustic frequency, a high frequency limit is imposed by the maximum size transform that can be handled by the computer code. The transform size is in turn limited by the available storage and computation time constraints. The time per range step is proportional to the transform size and hence to the frequency. Higher frequencies also demand smaller range steps. Thus, computation time for a given oceanographic environment is roughly proportional to the square of the frequency.

Fortunately in most ocean environments, long range low frequency sound propagation is dominated by rays at small grazing angles. Energy at steeper angles is lost due to bottom absorption and radiation. In these circumstances the parabolic equation technique is suitable for modeling the refracted and refracted surface-reflected paths that account for the propagation of energy to long ranges. Since the parabolic equation model is a wave model, diffraction and all other full wave effects are automatically included. In shallow water, however, or in the deep ocean when bottom interaction is important, the parabolic equation model in its present form is not appropriate.

## V THE COMPUTER CODE

The PE computer program consists of a main program driver and 14 computational subroutines. In addition, two environmental input subroutines must be supplied by the user. Output is written to an auxiliary (tape or disk) file as individual records containing the range in nautical miles and transmission loss (dB re 1 yard) at up to 20 output depths. Optional output includes a printed transmission loss table and a line printer field plot.

The user must supply two subroutines:

```
FORTRAN FUNCTION ZB(R)
FORTRAN SUBROUTINE SVP(NC,Z,C,RNEXT)
```

ZB returns the bottom depth in feet at range R in feet. SVP returns an NC point ($\leq$100) sound velocity profile table in arrays Z and C and RNEXT, the range of the next profile on the track in feet. Z is a depth array in feet or meters. C is a sound speed array in ft/sec or m/sec.

The core requirement, excluding the two user supplied environmental subroutines, is approximately 112,000 octal (38,000 decimal) on the CDC 6400/6600/6700.

The input deck structure can be summarized as follows:

Card 1 FORMAT (8A10)

Run title (80 columns text)

Card 2 FORMAT (5I5)

ND - Number of output depths $1 \leq ND \leq 20$

| | | |
|---|---|---|
| IFLAT | = 0 | Bottom is flat |
| | ≠ 0 | Bathymetry will be supplied by user FUNCTION ZB(R) |
| IPRNT | = 0 | No transmission loss table printed |
| | ≠ 0 | Print transmission loss table |
| NPLT | = 0 | No line printer field plot generated |
| | ≤ 120 | Generate line printer field plot of NPLT depths |
| $N<6$ | | Program will select transform size |
| $N \geq 6$ | | Program will run at specified transform size |

The current version of the program is dimensioned for $N \leq 11$. The maximum transform size is specified by $NMAX=2^N$ in SUBROUTINE PETL. NMAX must correspond to the dimensions of arrays in common blocks /FIELD/, /TABLE/, /WTS/ and arrays B, JI, and ST in SUBROUTINE RST.

Card 3 FORMAT (3F10.2)

ZS - Input depth in feet

F - Frequency in Hertz

$C_o$ - Reference sound speed

If $C_o \leq o$, the input environment will be transformed to reduce the parabolic phase velocity error.

If $C_o > o$ the specified reference sound speed will be used and the parabolic phase velocities will not be corrected.

Card 4 FORMAT (7F10.2)

DMAX - The maximum water depth on the track in feet

RMAX - The maximum range of the calculation in nautical miles

DR - The range step in nautical miles

If DR≤0, the code will select range steps aased on the split-step algorithm truncation error estimates. If DR>0, the code will run at the specified range step and all error checks are disabled.

CD1 - Minimum depth for the line printer field plot in feet

CD2 - Maximum depth for the line printer field plot in feet

CLMIN - Minimum loss for the line printer field plot in dB

DCL - Loss increment for the line printer field plot in dB

Card 5 FORMAT (8F10.2)

D(I) - ND output depths in feet

The formats for the environmental inputs (bathymetry and sound velocity profiles) will be determined by the user supplied subroutines.

The sequence of calculations proceeds essentially as follows:

1. The input data are read and printed in the main program driver.

2. SUBROUTINE PETL is called to compute the transmission loss.

3. SUBROUTINE PETL calls user supplied SUBROUTINE SVP to obtain the initial sound velocity profile. The depth mesh is determined if the transform size has not been specified

and relevant constants are computed. The transform size is checked to determine if array dimensions are exceeded. SUBROUTINE FILTER is called to evaluate the index of refraction on the depth mesh by linear interpolation in the sound speed profile table. If the parabolic phase velociiies are to be corrected the transformed mesh is generated. A 1-2-1 smoothing filter is applied on the index of refraction mesh to reduce transform aliasing. SUBROUTINE SOURCE is called to define the initial field as a Gaussian beam with the amplitude and standard deviation selected to match asymptotically the point source solution of the reduced wave equation in a homogeneous medium. The range loop is then initiated and SUBROUTINE STEP is called to advance the solution one range step.

4. If the range step is fixed SUBROUTINE STEP advances the solution the specified range step using the Tappert-Hardin split-step Fourier algorithm and returns to PETL. If the range step is not specified, every fifth call to STEP causes the sine transform to be examined for evidence of aliasing. If the ratio of energy in the last 1/4 of the spectrum to the total energy is greater than -20 dB, STEP will set the error return flag and a warning message will be printed upon returning to PETL. If aliasing is severe ($\geq$-14dB) the run will be terminated. On the other hand, if the spectral energy distribution permits, the

transform size will be reduced. STEP then computes a new range step at each mesh point using the split-step algorithm truncation error estimates. The mesh is searched for the minimum step size. If the minimum step is less than the acoustic wavelength, the error flag is set and a warning message will be printed upon returning to PETL. The program will then attempt to continue with the current range step. If the new range step is not small, and the relative change in step size is greater than 25%, STEP will replace the current range step with the new step, reconstruct all stored tables, and advance the solution.

5. Upon returning to PETL from STEP, the range is advanced and if required, a new sound velocity profile is obtained and filtered. Transmission loss at the output depths is interpolated from the field mesh and written to a tape or disk file. If requested (NPLT>0) the line printer field plot is generated. The error return flag from SUBROUTINE STEP is examined and if set, appropriate warning messages are printed and counted. The range loop will be continued until the maximum range is attained or five warnings have been issued. PETL then returns to the main program driver.

6. If requested (IPRNT≠0), the main program prints the transmission loss table and the run is terminated.

Small range step warning usually result from discontinuities or extreme gradients in the sound velocity

profile or an environment in which the sound velocity is not slowly varying in range. Large changes in the sound velocity profile between range steps cannot be handled by the numerical algorithm. Transform aliasing warnings indicate that the field is not adequately sampled in depth. The code attempts to select the proper transform size; however, since the sampling required depends on the solution, this is not always possible. If the code terminates because of transform aliasing, it is necessary to set the next larger transform size on card 1. The components of the computer code are documented in Appendix A.

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# APPENDIX A

## DESCRIPTION OF THE CODE COMPONENTS

## APPENDIX A

### PROGRAM PE

PE is the input/output driver for the parabolic equation model. It defines the FORTRAN logical units, reads and prints the input data, calls PETL to create a transmission loss file on FORTRAN Unit LT (disk or tape), checks for an error return from PETL, and prints an output transmission loss table if desired.

FORTRAN logical units LC, LP, and LT must be defined by the first three executable statements of PE. LC is the input device (card reader), LP is the output device (line printer), and LT is the file output device.

CARD INPUTS (FORTRAN Unit LC)

Card 1 - FORMAT (8A10)

TITLE - Run title (text 10 characters/word)

Card 2 - FORMAT (5I5)

ND - Number of output depths

IFLAT - Flat bottom flag

IPRNT - Print flag

NPLT - Number of line printer field plot depths

N - Transform size

Card 3 - FORMAT (3F10.2)

ZS - Input depth in feet

F - Frequency in Hertz

CO - Reference sound speed in ft/sec or m/sec

Card 4 - FORMAT (7F10.2)

DMAX - Maximum depth in feet

RMAX - Maximum range in nautical miles

DR - Range step in nautical miles

CD1 - Minimum field plot depth in feet

CD2 - Maximum field plot depth in feet

CLMIN - Minimum field plot loss in dB

DCL - Field plot loss increment in dB

Card 5 - FORMAT (8F10.2)

D(1) - ND output depths in feet

COMMON OUTPUTS

| | |
|---|---|
| /UNITS/ | LC - FORTRAN input unit |
| | LP - FORTRAN output unit |
| | LT - FORTRAN file output unit |
| /HERTZ/ | CO - Reference sound speed |
| | F - Frequency |
| /PLT/ | TITLE - Run title |
| | NPLT - Number of field plot depths |
| | CLMIN - Minimum loss |
| | DCL - Loss increment |
| | CD1 - Minimum plot depth |
| | CD2 - Maximum plot depth |
| /MESH/ | DR - Range step |
| | N - Transform size |

PRINTED OUTPUT (FORTRAN Unit LP)

PE prints the input data, and if IPRNT≠0, a range-transmission loss table for the output depths.

## SUBROUTINE PETL

PETL sets the maximum transform size, establishes the maximum depth sample in the transform, defines constants, sets up the initial conditions, controls the range loop, and creates an output transmission loss file. If the transform size has not been specified (N<6), PETL will select a transform size such that the depth mesh increment is less than the beam width of the Gaussian beam initial condition. If the reference sound speed has not been specified (CØ≤0), PETL sets the phase velocity correction flag to transform the environment to reduce the parabolic phase velocity error (MC-1), and takes the minimum sound speed as the reference sound speed. The dimensions of arrays in common blocks /FIELD/ and /TABLE/ must correspond to the maximum transform size established in PETL.

CALLING PROGRAM

PE

PARAMETER INPUTS

ZS - Input depth in feet

ND - Number of output depths

D - Output depths in feet

DMAX - Maximum depth in feet

RMAX - Maximum range in feet

IFLAT - Flat bottom flag

COMMON INPUTS

| | | |
|---|---|---|
| /UNITS/ | LP - FORTRAN output unit |
| | LT - FORTRAN file output unit |
| /HERTZ/ | CO - Reference sound speed |
| | H - Mesh increment in transform space |
| | HK - Ratio of mesh increment in transform space to reference wave number |
| | FK - Reference wave number |
| | WL - Acoustic wavelength |
| /PLT/ | DCD - Field plot depth increment |
| | CD - Field plot depth array |
| /MESH/ | R - Current range in feet |
| | NR - Range step count |
| | KR - Flagged step count |
| | DZ - Depth mesh increment in feet |
| | ZMAX - Maximum depth sample in the transform in feet |
| | IB - Depth index of bottom interface |
| | N - Transform size |
| | NPTS - Number of field mesh points ($2^N$-1) |
| | N2 - 1/2 the number of field mesh points |
| | N4 - 1/4 the number of field mesh points |
| | NL4 - 3/4 the number of field mesh points |

NA - Number of points in the artificial attenuation table

NW - Number of points in the water column

ZW - Current bottom depth in feet

HALF - Half the number of field mesh points

/PHASE/ NC - Number of points in current sound speed profile

Z - Sound speed depth array

C - Sound speed array

MC - Phase velocity correction flag

DM - Transformed output depths

/OUTBUF/ RNM - Current range in nautical miles

TL - Transmission loss at the output depths in dB

FILE OUTPUT

Unit LT - One record for each range step including the range in nautical miles and the transmission loss at up to 20 output depths

PRINTED OUTPUT

PETL prints the transform size and appropriate warnings if the error return flag from STEP is set.

## FUNCTION TLOSS

TLOSS interpolates the absolute value of the field at depth Z from the field mesh and returns the transmission loss. The interpolation is linear in the absolute value

of the field. The maximum loss permitted is 180 dB.

CALLING PROGRAMS

PETL

FLD

PARAMETER INPUTS

RR - Reciprocal range in inverse feet

Z - Depth in feet

COMMON INPUTS

| | |
|---|---|
| /FIELD/ | PR - Real part of the field mesh |
| | PI - Imaginary part of the field mesh |
| /MESH/ | DZ - Depth mesh increment in feet |

FUNCTION OUTPUT

TLOSS - Transmission loss at depth Z

## SUBROUTINE SOURCE

SOURCE constructs the field to be used as an initial condition for the numerical integration. The field is defined as a Gaussian beam at zero range. The parameters of the beam are selected to match asympotically the point source solution of the elliptic wave equation. The width, GW, is given by

$$GW = \frac{2}{FK}$$

where FK is the average wave number. The amplitude, GA, is given by

$$GA = \frac{1}{GW}\left(\frac{2}{FK}\right)^{1/2}$$

The real part of the field, PR, is set equal to the beam minus its image.

$$PR = GA\left[e^{-\left(\frac{ZM-ZS}{GW}\right)^2} - e^{-\left(\frac{-ZM-ZS}{GW}\right)^2}\right]$$

where ZM is the depth mesh point and the exponent is constrained to be greater than -42. The imaginary part of the field is set to zero.

PI = 0

<u>CALLING PROGRAMS</u>

PETL

<u>PARAMETER INPUTS</u>

ZS - Input depth in feet

<u>COMMON INPUTS</u>

| | | |
|---|---|---|
| /HERTZ/ | FK | - Reference wave number |
| /MESH/ | DZ | - Depth mesh increment in feet |

<u>COMMON OUTPUTS</u>

| | | |
|---|---|---|
| /FIELD/ | PR | - Real part of the initial field |
| | PI | - Imaginary part of the initial field |

## FUNCTION SPEED

SPEED linearly interpolates the sound speed at depth D

### CALLING PROGRAMS

PETL

FILTER

### PARAMETER INPUT

D - Depth in feet

### COMMON INPUTS

/PHASE/ NC - Number of sound velocity profile points

Z - Depth array

C - Sound speed array

### FUNCTION OUTPUT

SPEED - Sound speed at depth D

## SUBROUTINE FILTER

FILTER evaluates the index of refraction on the field mesh and transforms the environment to reduce the phase velocity error inherent in the parabolic approximation. The input sound speed profile is checked and if required the units are converted to feet and ft/sec. The sound speed is assumed to be piecewise linear in depth. The output array FN contains $n^2-1$ on a uniform mesh where n is the index of refraction or the transformed index of refraction

$$\tilde{n}=(Zn-1)^{1/2}$$

if the parabolic phase velocity correction is applied. The output array is filtered to smooth the gradient discontinuities introduced by the boundaries in the input profile. The transformed depths are given by

$$\tilde{z} = z\left[\frac{C_o}{C(z)}\right]^{1/2}$$

The inverse mapping

$$Z = \frac{g\tilde{Z}^2+\tilde{Z}\sqrt{(g\tilde{Z})^2+4C_o(C_k-gZ_r)}}{2C_o}$$

where g is the sound speed gradient

$Z_k$ is the boundary depth

$C_k$ is the sound speed at the boundary depth

is used to evaluate the sound speed on the transformed depth mesh. The flag MC is used to signal the pahse velocity correction mapping.

MC = 1 Phase velocity correction

MC = 2 No phase velocity correction

CALLING PROGRAMS

PETL

PARAMETER INPUTS

ND - Number of output depths

D - Output depth array

COMMON INPUTS

| | | |
|---|---|---|
| /UNITS/ | LP - FORTRAN output unit |
| /HERTZ/ | CO - Reference sound speed in ft/sec |
| /PHASE/ | NC - Number of sound speed profile points |
| | Z - Depth array |
| | C - Sound speed array |
| | MC - Phase velocity correction flag |
| /PLT | NPLT - Number of field plot depths |
| | CD1 - Minimum field plot depth |
| | DCD - Field plot depth increment |
| /MESH/ | DZ - Depth mesh increment in feet |
| | NW - Maximum number of points in the water column |
| | NPTS - Number of field mesh points |

COMMON OUTPUTS

| | |
|---|---|
| /PHASE/ | DM - Transformed output depths |
| /PLT/ | CD - Transformed field plot depths |
| /TABLE/ | FN - Filtered index of refraction array |

PRINTED OUTPUT

Warning message if the sound speed profile is not defined to the surface. In this case the run is aborted.

## SUBROUTINE SET

SET constructs all tables that are a function of the range step. The artificial bottom attenuation table is of the form

$$A(I) = e^{-0.01*DR*e^{-\left(\frac{DZ*I-DZ*NA}{1/3\quad DZ*NA}\right)^2}}$$

where DZ is the depth increment in feet

DR is the range step in feet

NA is the number of points in the table

The second derivative transform table is of the form

$$S = \frac{e^{-i\frac{DR}{2F_k}K^2}}{2^{N-1}}$$

where DR is the range step in feet

FK is the average wave number

K is the spacial Fourier transform variable from Z to K

N is the transform size

The factor $2^{N-1}$ provides the transform normalization. The real part is returned in array SR. The imaginary part is returned in array SI.

SUBROUTINE INDEX is called to generate an index of refraction table of the form

$$U = e^{i \frac{DR*FK}{2} (n^2-1)}$$

where DR is the range step in feet

FK is the average wave number

n is the index of refraction

The real part is returned in array UR. The imaginary part is returned in array UI.

CALLING PROGRAMS

PETL

STEP

COMMON INPUTS

| | | |
|---|---|---|
| /HERTZ/ | H - Mesh increment in transform space |
| | FK - Reference wave number |
| /MESH/ | DR - Range step in feet |
| | DZ - Depth increment in feet |
| | NPTS - Number of depth mesh points |
| | NA - Number of points in the attenuation table |
| | HALF - $2^N-1$ where N is the transform size |

COMMON OUTPUTS

| | |
|---|---|
| /TABLE/ | A - Attenuation table |
| | SR - Real part of the transform table |
| | SI - Imaginary part of the transform table |
| | UR - Real part of the index of refraction table |

UI - Imaginary part of the index of refraction table

/HERTZ/ FACTOR - Constant for generating index of refraction table (DR*FK/2)

## SUBROUTINE INDEX

INDEX constructs an index of refraction trable of the form

$$U = e^{\frac{DR*FK}{2}(n^2-1)}$$

where DR is the range step

FK is the reference wave number

n is the index of refraction

The real part of the table is returned in array UR. The imaginary part is returned in array UI. The artificial attenuation is incorporated in the output table

CALLING PROGRAMS

PETL

SET

COMMON INPUTS

/HERTZ/ FACTOR - DR*FK/2 where DR is the range step and FK is the reference wave number

/MESH/ NPTS - Number of depth mesh points

NW - Maximum number of points in the water column

NA - Number of points in the attenuation table

IB - Mesh index for start of bottom

/TABLE/ A - Attenuation table

COMMON OUTPUTS

/TABLE/ UR - Real part of index of refraction table

UI - Imaginary part of index of refraction table

## SUBROUTINE STEP

STEP advances the field one range step using the Tappert-Hardin split-step Fourier algorithm. If the range step has not been fixed, STEP will perform error checks and use the numerical algorithm trunfation error estimates to compute a new range step every fifth call (i.e. when NR=KR). If the relative change in step size is greater than 25%, the current range step is updated and new tables are constructed. If the predicted range step is less than the acoustic wavelength or the ratio of the energy in the last 1/4 of the spectrum to the total energy is greater than -20 dB (evidence of transform aliasing), the error return flag is set (FLAG = 0). In addition two other spectral energy ratios are examined and if all criteria are met, the field is considered to be oversampled in depth and the transform size is reduced.

CALLING PROGRAMS

PETL

COMMON INPUTS

/FIELD/ PR - Real part of field at range R

PI - Imaginary part of field at range R

| | | |
|---|---|---|
| /HERTZ/ | H - Mesh increment in transform space | |
| | HK - Ratio of mesh increment in transform space to the reference wave number | |
| | FK - Reference wave number | |
| | WL - Acoustic wavelength | |
| /MESH/ | DR - Current range step in feet | |
| | NR - Current range step count | |
| | KR - Current flagged step count | |
| | DZ - Depth mesh increment in feet | |
| | IB - Mesh index for start of bottom | |
| | N - Transform size | |
| | NPTS - Number of field mesh points | |
| | N2 - 1/2 the number of field mesh points | |
| | N4 - 1/4 the number of field mesh points | |
| | NL4 - 3/4 the number of field mesh points | |
| | NA - Number of attenuation table points | |
| | NW - Number of mesh points in the water column | |
| | HALF - 1/2 the number of field mesh points | |
| /TABLE/ | A - Artificial attenuation table | |
| | FN - $n^2$-1 where n is the index of refraction | |
| | SR - Real part of transform table | |
| | SI - Imaginary part of transform table | |
| | UR - Real part of index of refraction table | |
| | UI - Imaginary part of index of refraction table | |

PARAMETER OUTPUTS

FLAG - Error return flag

COMMON OUTPUTS

| | | |
|---|---|---|
| /FIELD/ | PR - Real part of field at range R+DR | |
| | PI - Imaginary part of field at range R+DR | |
| /MESH/ | DR - Updated range step | |
| | KR - Updated flagged step count | |
| | DZ - Depth mesh increment | |
| | N - Transform size | |
| | NPTS - Number of field mesh points | |
| | N2 - 1/2 the number of field mesh points | |
| | N4 - 1/4 the number of field mesh points | |
| | NL4 - 3/4 the number of field mesh points | |
| | NA - Number of points in the attenuation table | |
| | NW - Maximum number of mesh points in the water column | |
| | HALF - 1/2 the number of field mesh points | |
| /TABLE/ | A - Artificial attenuation table | |
| | FN - $n^2-1$ where n is the index of refraction | |
| | SR - Real part of the transform table | |
| | SI - Imaginary part of the transform table | |
| | UR - Real part of the index of refraction table | |
| | UI - Imaginary part of the index of refraction table | |

## FUNCTION GAUSS

GAUSS evaluates the Gaussian function

$$GA * e^{-\left(\frac{Z-CS}{GS}\right)^2}$$

The exponent is not allowed to be less than -42.

CALLING PROGRAMS

SET

SOURCE

PARAMETER INPUTS

GA - Gaussian amplitude

GD - Gaussian depth in feet

GW - Gaussian width in feet

Z - Depth in feet

FUNCTION OUTPUT

$$GAUSS = GA * e^{-\left(\frac{Z-GD}{GW}\right)^2}$$

## SUBROUTINE FLD

FLD is a routine for plotting the transmission loss field generated by the parabolic equation model on the line printer. Symbols corresponding to five transmission loss bins at up to 120 depths are plotted at approximately one nautical mile range increments. If the plot depth is greater than the current water depth, B is printed. A transmission loss scale and a depth scale are printed the first time FLD is called.

CALLING PROGRAMS

PETL

PARAMETER INPUTS

| | |
|---|---|
| | RR - Reciprocal range |

COMMON INPUTS

| | |
|---|---|
| /UNITS/ | LP - FORTRAN output unit |
| /OUTBUF/ | RNM - Current range in nautical miles |
| /PLT/ | TITLE - Run title (text) |
| | NPLT - Number of plot depths |
| | LCR - Last range plotted |
| | CLMIN - Minimum plot loss in dB |
| | DCL - Loss increment in dB |
| | CD1 - Minimum plot depth in feet |
| | DCD - Plot depth increment in feet |
| | CD - Plot depth array in feet |
| /MESH/ | R - Current range in feet |
| | DR - Current range step in feet |
| | ZW - Current water depth in feet |

COMMON OUTPUTS

| | |
|---|---|
| /PLT/ | LCR - Current plot range |

PRINTED OUTPUT

Line printer plot of current range and transmission loss bins at up to 120 depths.

## SUBROUTINE RST

RST is a sine transform driver for a real vector mixed radix (8-4-2) fast Fourier synthesis algorithm developed by Bergland.[8] The sine transform algorithm is from Cooley, Lewis and Welsh.[9] RST defines constrants, sets up the necessary tables, forms the input Fourier coefficients for the fast Fourier synthesis subroutines, and forms the sine transform from the results. Upon returning from RST the real input vector has been replaced by its finite discrete sine transform. The dimensions of arrays in RST must correspond to the largest vector to be transformed.

| ARRAY | REQUIRED DIMENSIONS |
|---|---|
| B | $2^n$ |
| ST | $2^n-1$ |
| JI | $2^{n-1}-1$ |
| CS | $2^{n-4}-1$ |
| SS | $2^{n-4}-1$ |

where the real input vector is of dimension $2^n-1$

### CALLING PROGRAMS

STEP

### PARAMETER INPUTS

X - Real vector

N - Transform size (input vector of size $2^N-1$)

### PARAMETER OUTPUTS

X - Sine transform of the input vector

COMMON OUTPUTS

/WTS/

NT - Size of trigonometric table for a given radix 8 iteration

CS - Cosine table stored in bit-reversed order

SS - Sine table stored in bit-reversed order

SUBROUTINE R8SYN

R8SYN performs one pass of a radix 8 real vector fast Fourier synthesis.

CALLING PROGRAM

RST

PARAMETER INPUTS

INT - Length of current pass

B0

B1

B2

B3 - Input vectors for current radix 8 pass

B4

B5

B6

B7

COMMON INPUTS

/WTS/

NT - Length of trigonometric table

CS - Cosine table stored in bit-reversed order

SS - Sine table stored in bit-reversed order

PARAMETER OUTPUTS

B0

B1

B2

B3

B4 - Output vectors of current radix 8 pass

B5

B6

B7

## SUBROUTINE R4SYN

R4SYN performs a single pass radix 4 real vector fast Fourier synthesis.

CALLING PROGRAMS

RST

PARAMETER INPUTS

INT - Length of radix 4 pass

B0

B1

B2 - Input vectors for radix 4 pass

B3

PARAMETER OUTPUTS

B0

B1

B2 - Output vectors from radix 4 pass

B3

## SUBROUTINE R2TR

R2TR performs a single pass radix 2 fast Fourier transform.

CALLING PROGRAM

RST

PARAMETER INPUTS

INT - Length of radix 2 pass

B0, B1 - Input vectors for radix 2 pass

PARAMETER OUTPUTS

B0, B1 - Output vectors from radix 2 pass

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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The Parabolic Equation Model is a wave-acoustics model designed for the computation of acoustic transmission loss as a function of range and depth in range dependent ocean environments. The elliptic reduced wave equation is approximated by a parabolic partial differential equation that can be numerically integrated by marching the solution forward in range. The model is primarily useful for predicting low frequency (< 200 Hz)

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acoustic propagation of energy along waterborne paths. This report briefly describes the physics and mathematics of the model and documents a computer program developed at AESD. Individual routines are documented in an appendix. Environmental input routines must be supplied by the user and are not described in this report.